Article

Game Analysis of Government Regulation and Private Equity Investment Funds

Yonghong Yu[1], Li Wu[2], Guannan Wei[1]

1 School of Management Science and Engineering, Anhui University of Finance & Economics, Bengbu 233030, China;
2 School of Finance and Public Management, Anhui University of Finance & Economics, Bengbu 233030, China.

Abstract: The relevant entities involved in private equity investment funds include investors, private equity investment funds, and financing enterprises. With the policy background of establishing a multi-level financial market, there exists significant risks in the operation of PE funds at all stages, it is necessary to introduce government regulatory role in the operation of PE funds to reduce risks. This paper mainly discusses about the coordination between the government and private equity investment funds from the perspective of game theory, it provides a complete information static game theory analysis of the government and private equity investment funds and gives the mixed strategy Nash equilibrium. It also proposes some suggestions to protect the rights and interests of investors and ensure the stability of the national financial system.

Keywords: Private Equity Investment Funds; Game Theory; Mixed Strategy Nash Equilibrium; Government Regulation

1. Introduction

As a production of the combination of financial innovation and industrial innovation, private equity(PE) funds play an important role in providing a valuable capital and sustainable development for start-up companies, spawning new industries, accelerating the upgrade of industrial structure and consumption level, and creating more employment opportunities. Due to the characteristics of non-disclosure and non-listing, the trading process and details of private equity investments are rarely disclosed publicly, and this uncertainty makes it difficult for investors to make investment decisions, and the management of private equity investment funds also poses significant risks. In addition, the

dual principal-agent game relationship and the long-term and poor liquidity characteristics of the investment cycle make it more difficult to control risks.

There exist a number of researches on the strategies and methodologies of private equity [1-5], Cumming [1] proposed that effective and reasonable government regulations and supervision can guide institutional investments to increase the asset allocation ratio of private equity investments by studying the issue of information disclosure in PE investments. Conversely, the lack of a macro environmental system can have a negative impact. Heed [2] pointed out that non systematic risks caused by the unique characteristics of private equity investments generate market fluctuations, and it is necessary to establish sound regulatory systems in the market to prevent them. Millner [3] proposed to study the information disclosure norms of industry organizations such as EVCA and NVCA, as well as the information disclosure behavior of the PE industry, including value evaluation, degree of information disclosure, performance forecasts, and other content. These contents require a set of standard industry rules and legal systems, which are jointly executed and complied with by relevant participating parties. Wu [4] believed that China needs to further recognize the legal status of the development of private equity investment funds at the legal level. Currently, it is urgent to enrich China's multi-level capital market to cooperate with the growth of private equity investment funds. Zhou [5] took the economic development status after the financial crisis as the research object, combined with the current government regulatory situation in China, and pointed out that China should appropriately learn from overseas regulatory experience, and based on China's national conditions, study a regulatory system that truly conforms to China's characteristics of private equity investment funds.

At present, a large number of studies on private equity investment funds based on game theory and contract theory [6-9], involving various stages of private equity investment, such as project evaluation and exit models, but there are few researches focusing on the relationship between private investment funds and government regulation strategies. Although investors, private equity investment funds, and financing enterprises are the three major participants in the investment operation process, the government regulatory agencies are also important participants in the process of private equity investment, playing an irreplaceable role in regulating strategic direction and maintaining the healthy and stable development of the market. This article constructs a game model between government regulatory agencies and private equity investment funds, and studies relevant strategies at the government regulatory level, hoping to protect the rights and interests of investors and ensure the stability of the national financial system.

2. Preliminary

Game theory is the formal study of decision-making in which economic agents make strategic interactions to produce outcomes to maximize their own utility under certain constraints. According to Gibbons [10], Zhang [11] and Nisan [12], there exist following basic concepts and theorem:

Definition 1 Given the *n*-player game $G = \{S_1,...,S_n; u_1,...,u_n\}$, the strategies $\{S_1^*, ..., S_n^*\}$ is a Nash equilibrium if, for each player *i*, s_i^* is player *i*'s best response to the strategies of the *n-1*

other players $\{S_1^*, \ldots, S_{i-1}^*, S_{i+1}^*, \ldots, S_n^*\}$, $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \forall s_i \in S_i, \forall i$ for every

feasible strategy s_i in S_i, that is, $s_i{}^*$ solves $\max\limits_{s_i \in S_i} u_i(s_1^*, \ldots, s_{i-1}^*, s_i^*, s_{i+1}^*, \ldots, s_n^*), i = 1, 2, \ldots, n$.

Definition 2   Given the n-player game $G = \{S_1, \ldots, S_n; u_1, \ldots, u_n\}$, for each player i,

$S_i = \{s_{i1}, \ldots, s_{ik}\}$, Then a mixed strategy for player i is a probability distribution

$p_i = \{p_{i1}, \ldots, p_{ik}\}$, where $k = 1, \ldots K, 0 \leq p_{ik} \leq 1, \sum_i^k p_{ik} = 1$.

Definition 3   Given the *n*-player game $G = \{S_1, \ldots S_n; u_1, \ldots, u_n\}$, the mixed strategies

$p^* = \{p_1^*, \ldots, p_i^*, \ldots, p_n^*\}$ is a Nash equilibrium if $v_i(p_i^*, p_{-i}^*) \geq v_i(p_i, p_{-i}^*), \forall p_i \in \sum_i$ for each

player *i=1,2,...,n*.

Theorem 1 In the *n*-player game $G = \{S_1, \ldots, S_n; u_1, \ldots, u_n\}$, if *n* is finite and S_i is finite for

every *i*, then there exists at least one Nash equilibrium, possibly involving mixed strategies.

3. Game model analysis between the government and private equity investment funds

Without loss of generality, there exists a 2-players game, and all players of this game are assumed to be rational and risk neutral, and this is common knowledge, we can represent the Normal form of 2-player complete information static game may as follow:

(1) Player set: defined as *N = {1,2}*, each of them represents a player in game, here 1 means the government, 2 means the private equity fund.

(2) Strategy set: defined as s_1 = *{supervise, no- supervise}*, s_2 = *{honesty, violation}*. Strategy s_{11}= *supervise*, s_{12}= *no-supervise*, s_{21}= *honesty*, s_{22}= *violation*.

(3) Payoff function: defined as *u1(s1j, s2j)* and *u2(s1j, s2j)* as the payoff of the government and the private equity fund respectively, which can be expressed as follows:

$u_1(s_{11}, s_{21}) = u-c2$, $u_1(s_{11}, s_{22}) = u-c1+\alpha f$, $u_1(s_{12}, s_{21}) = u$, $u_1(s_{12}, s_{22}) = u-\beta c3$

$u_2(s_{11}, s_{21}) = r$, $u_2(s_{11}, s_{22}) = r +y - a(x + y)$, $u_2(s_{12}, s_{21}) = r$, $u_2(s_{12}, s_{22}) = r +y-\beta(x + y)$

Where *r* denotes the total utility of standardized operation of private equity investment funds, *x* denotes difference between the total utility of private equity investment funds that operate in violation of regulations and are effectively regulated by regulatory agencies and the utility of standardized operation, *y* denotes the difference between the total utility obtained by private equity investment funds operating in violation of regulations and not being effectively regulated by regulatory agencies and the utility obtained by standardized operation, *x+y* denotes the punishment intensity of government regulation, *u* denotes the effectiveness of effective supervision by government regulatory agencies, *c1* denotes the cost of government regulatory agencies regulating violations of private equity investment funds, *f* denotes the effectiveness of regulatory agencies in effectively regulating the illegal behavior of private equity investment funds, *c2* denotes the cost of private equity investment funds regulated and operated by regulatory agencies, *c3* denotes the utility loss caused by insufficient self-regulation in the industry. If the probability of government regulation being effective for non-compliant private equity investment funds is *α*, then the probability of non-effectiveness for non-compliant private equity investment funds is *1-α*. If the probability of private equity investment funds operating illegally in government regulatory gaps and failing to evade third-party industry self-regulation is *β*, then the probability of avoiding

industry self-regulation during the illegal operation process is *1-β*. This game can be represented in the payoff matrix in Table 1:

Table 1. The Payoff Matrix of Mixed Strategies

		The private equity fund	
		honesty	violation
The government	Supervice	$u-c2$，r	$u-c1+\alpha f$，$r+y-\alpha(x+y)$
	No-supervice	u，r	$u-\beta c3$，$r+y-\beta(x+y）$

There are 2 players and each player has only two strategies, both of them are finite. As the game model mentioned above, there are four pure strategies that may achieve equilibrium, namely *{supervise, honest}, {no-supervise, honest}, {supervise, violation}, and {no-supervise, violation}*, each condition for achieving equilibrium of these four strategies can be presented as follow:

(1) If the equilibrium solution of the game is *{supervise, honest}*, the private equity investment funds operate in a standardized manner while government regulatory agencies regulate the industry. The conditions for achieving equilibrium in this strategy are:

$$u - c2 > u - \beta c3 \text{ and } r > r + y - \alpha(x + y), \text{ that is } \beta > \frac{c2}{c3} \text{ and } \alpha > \frac{y}{x+y}$$

(2) If the equilibrium solution of the game is *{no-supervise, honest}*, the private equity investment funds choose to operate in a standardized manner, and there is a gap in government regulation. The conditions for achieving equilibrium in this strategy are:

$$r + y - \beta(x + y) < r \text{ and } u - c1 + \alpha f < u \text{ , that is } \beta > \frac{y}{x+y} \text{ and } \alpha f < c1$$

(3) If the equilibrium solution of the game is *{no-supervise, violation}*, the private equity investment funds choose to operate in violation of regulations while government regulation is lacking. The conditions for achieving equilibrium in this strategy are:

$$r + y - \beta(x + y) > r \text{ and } u - \beta c3 > u - c1 + \alpha f \text{ , that is } \beta < \frac{y}{x+y} \text{ and } \frac{\alpha f}{c1 - \beta c3} < 1$$

(4) If the equilibrium solution of the game is *{supervise, violation}*, the private equity investment funds choose to operate illegally while government regulatory agencies conduct industry regulation. The conditions for achieving equilibrium in this strategy are:

$$r + y - \alpha(x + y) > r \text{ and } u - c1 + \alpha f > u - \beta c3, \text{ that is } \alpha < \frac{y}{x+y} \text{ and } \frac{\alpha f}{c2 - \beta c3} < 1$$

According to the equilibrium analysis of pure strategy, when the conditions *β<y/(x+y), α>y(/x+y), and αf/(c1-βc3)>1* are met, the operation process of private equity investment funds will fall into a cycle of "standardized operation of private equity investment funds -> government regulatory clearance -> illegal operation of private equity investment funds -> government regulatory implementation -> standardized operation of private equity investment funds", and at this point, a mixed strategy can be used to solve the equilibrium solution.

According to theorem 1, there exists a Nash equilibrium of mixed strategy. Assume the private equity fund selects honesty strategy in probability *a* and violation strategy in probability *1-a*. The government selects supervice strategy in probability *b* and no-supervice

strategy in probability *1-b*. Then, the expected payoff function of both players can be represented as follows:

$$U_{Pe} = a[b(r + y - \alpha x - \alpha y) + (1 - b)(r + y - \beta x - \beta y)] + (1 - a)r \quad (1)$$

$$U_g = b[a(u - c1 + \alpha f) + (1 - a)(u - c2)] + (1 - b)[a(u - \beta c3) + (1 - a)u] \quad (2)$$

The first order partical derivative of the expected payoff function with respect to independent variable *a, b* is:

$$\frac{dU_{Pe}}{da} = y - \beta(x + y) - b(x + y)(\alpha - \beta) = 0$$

$$\frac{dU_g}{db} = a(\alpha f + c2 + \beta c3 - c1) - c2 = 0$$

We can obtain the mixed strategies Nash equilibrium as:

$$b^* = \frac{y - \beta(x + y)}{(x + y)(\alpha - \beta)} \quad (3)$$

$$a^* = \frac{C2}{\alpha f + C2 + \beta C3 - C1} \quad (4)$$

When the regulatory probability of government regulatory agencies is greater than *b**, the optimal behavior of private equity investment funds is to choose standardized operation. When the regulatory probability of government regulatory agencies is less than *b**, the optimal behavior of private equity investment funds is to choose to operate illegally. When the regulatory probability of government regulatory agencies is equal to *b**, private equity investment funds achieve the same utility whether they choose to operate in a standardized manner or in violation of regulations.

When the probability of private equity investment funds operating illegally is greater than *a**, the optimal behavior of government regulatory agencies is to choose to regulate. When the probability of private equity investment funds operating illegally is less than *a**, the optimal behavior of government regulatory agencies is to choose regulatory short selling. When the probability of private equity investment funds operating in violation of regulations is equal to *a**, government regulatory agencies have the same effect whether they choose to regulate or non-regulate strategy.

4. Suggestions

As mentioned above, many factors are related to the Nash equilibrium of game among all players. In order to ensure the smooth development of private equity funds, it is necessary to design an effective incentive and punishment mechanism to ensure that all players perform positive behaviors.

Suggestion 1: Establishing and improving laws and regulations by government regulatory agencies with moderate supervision. According to the equations (1) - (4), it shows that establishing sound laws and regulations by government regulatory agencies can increase the probability of government regulatory agencies' supervision, thereby promoting the standardized operation of private equity investment funds and ensuring the healthy and stable development of the private equity investment market. At the same time, blind intervention should not be allowed, and moderate supervision should be achieved.

The primary goal of fund regulation is to protect the interests of investors. Investors are the support of the fund market, but they are in a disadvantaged position in the fund market. Compared to fund managers, investors often lack professional knowledge, insufficient access to information, weak risk identification and tolerance, and their legitimate rights and interests are easily violated. Therefore, fund supervision must effectively protect investors' legitimate rights and interests, so that investors can avoid unfair treatment. At the same time, fund supervision should also protect the legitimate rights and interests of relevant parties in the fund market in accordance with the law. Regulating the activities of private equity investment funds is an inevitable requirement for the regulatory goal of protecting the legitimate rights and interests of investors and related parties. Regulating the activities of private equity investment funds is the main means and institutional guarantee for protecting the legitimate rights and interests of investors.

For funds, the scope of government regulation should be strictly limited to the areas where the fund market fails. The self-discipline mechanism of the fund industry should be improved, and the internal control and supervision mechanism of fund institutions should be strengthened to cultivate social forces. The active role of self-discipline in the fund industry, internal control of fund institutions, and social force supervision in fund regulation should be fully utilized, forming a system with government regulation as the core, industry self-discipline as the link, and institutional internal control as the foundation The "Four in One" regulatory pattern supplemented by social supervision.

Suggestion 2: Strengthing industry self-discipline and supervision. The larger x, the smaller y and the larger β in equations (3), the more perfect the third-party self-discipline environment and the stronger the self-discipline ability, and the less likely private equity investment entities are to operate in violation of regulations. The less personal benefits that can be obtained from illegal operations, the heavier the punishment for private equity investment entities who are punished for illegal operations, and the less motivation private equity investment entities have to choose illegal operations.

According to the game model analysis between the government and private equity investment funds mentioned above, due to the existence of asymmetric and incomplete information between the two parties, on the one hand, as long as the government regulatory agencies effectively regulate and pay high regulatory costs, it will promote the standardized operation of private equity investment main funds. When the utility of standardized operation is lower than that of illegal operation, there is a possibility of illegal operation in private equity investment funds. On the other hand, from the perspective of government regulatory agencies, when regulatory costs are high and regulatory effectiveness is negative, there is a possibility for regulatory agencies to abandon regulation. At this point, the existence of third-party self-regulatory organizations is of great significance for the healthy development of the industry.

5. Conclusions

This paper discusses the coordination between the government and private equity investment funds under the view of game theory. The aim of this article is to design a rational

mechanism that can reduce risks of PE and also obtain the outcome of mixed strategy Nash equilibrium. According to the analysis result mentioned above, we design a mechanism that can provide the effective cooperation between government regulatory agencies and private equity investment funds. Based on China's national conditions and development status, we should establish and improve laws and regulations by government regulatory agencies with moderate supervision, and strengthen industry self-discipline and supervision at the government regulatory level.

References:

1. Cumming D, Johan S. Regulatory harmonization and the development of private equity markets. Journal of Banking＆Finance. 2007; 31(10):3218-3250.

2. Heed A. Regulation of private equity. Journal of Banking Regulation. 2010; 12(1):24-47.

3. Millner, S. Transparency as a competitive tool in the spotlight. New York, Boston-London, 2002,17.

4. Wu Xiaoling. Researches on several issues of development of private equity funds. China Finance. 2007; 11:10-12.

5. Zhou Yingying. Characteristics and problem analysis of private equity fund regulation after crisis. Legal System and Society. 2012; 30:102-104.

6. Chan Y. On the positive role of financial intermeation in allocation of venture capital in a market with imperfect information. Journal of Finance. 1983; 38:1543-1568.

7. Adman A. and Pfleiderer P. Robust financial contracting and the role for venture capitalists. Journal of Finance. 1994; 49:371-402.

8. Elitzur R. and Gavious A. Multi-period games with repeated contracts between venture capitalists and entrepreneurs. University of Toronto workingpaper, 2001.

9. Wang Yunfeng. Game analysis of private equity investment. Capital University of Economics and Business, China, 2015.

10. Gibbons, R. A primer in game theory. Prentice Hall, 1992.

11. Zhang, W. Game theory and information economics. Shanghai People's Publishing House, China, 2002.

12. Nisan, N. Algorithmic game theory. Cambridge University, England, 2007.